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EFFECT OF SCATTERING ON THE CONTOUR OF THE RADIATION LINE OF A PLANE LAYER OF A SELECTIVE MEDIUM

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An analytical expression for determination of the correction factor which allows for effect of scattering on the contour of the radiation line of a plane layer of a selective medium is presented. Errors of calculation of the intensity of radiation which arise when this effect is neglected are analyzed. The study is based on a numerical solution of the integro-differential transfer equation.

The rapid development of technology, the continuously expanding use of high-temperature techniques, the increase in the power of energy units, and the new engineering possibilities of remote, nondestructive testing of technical and natural media pose new, stricter requirements on the rate and accuracy of calculation of characteristics of radiation transfer in absorbing, radiating, and scattering media; this makes it necessary to more correctly allow for such factors as the selectivity of a gas medium, the multiplicity of the processes of radiation scattering, etc. The latter presupposes further study of this problem.

In the presence of macroscopic particles, the radiation of a selective gas medium can change drastically due to natural radiation and an increase in total optical thickness. This fact leads to difficulties associated with localization of the volume of observation. If the number of particles in the medium is rather large, the effect of scattering can be considerable even in the absence of external radiation. Therefore, account for scattering of radiation in diagnostics of a selectively radiating gas with a condensed disperse phase is a rather complex problem.

The solution of the integro-differential equation of radiation transfer, which forms the basis of theoretical studies of these problems, is associated, even in the case of a "grey" gas, with certain mathematical difficulties, especially for multidimensional geometry and a nonisothermal medium. The introduction of integration with respect to frequency makes this problem virtually unsoluble.

However, it is necessary to allow for the effect of scattering processes on the shape of the line, because in a number of cases it could be very substantial. Scattering of radiation on particles of a condensed phase can decrease or increase the emerging radiation. It is shown in [1] that the optical properties of the particles play determining role.

If we pass from integration with respect to frequency to integration with respect to the coefficient of absorption, having determined the distribution function of it, then for some models of gas lines we are able to integrate the equation of radiation transfer [2, 3]. Here, in fact, we need only one integration with respect to the coefficient of absorption instead of integration with respect to many lines. The distribution function of the coefficient of absorption for the Malkmus and Goody models was obtained in [2]. But its use for direct integration of the equation of transfer is difficult, due to some of its special features and the impossibility of tracing changes in the contour of the line in the presence of scattering.

The problem of transfer of selective radiation in the presence of a condensed phase in MHD generators was solved by the Sobolev method in [4-6]. The increase in intensity on the wings of lines under the effect of the processes of scattering on solid particles is shown by the examples of the lines of cesium and sodium.

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In [7] a method of averaging over the spectrum of lines and molecular bands with account for scattering in a nonuniform medium for the H_2O band (4014–4018 cm) is suggested, and it is shown that calculation for "grey" radiation can increase errors by up to 200% A solution of the equation of transfer was obtained in a two-flow approximation, which itself gives large errors in calculation of the characteristics of transfer and, consequently, is of little use for these studies.

On the basis of a two-flow solution of the equation of an exponential model for molecular bands with use of the distribution function of the coefficient of absorption from [2], a relatively simple expression for hemispherical emissivity of a plane line is obtained in [8]. The emissivity of the layer as a function of the optical thickness of the layer is analyzed in the same paper. However, due to insufficient accuracy of the equation of transfer, this analysis gives a qualitative rather than a quantitative estimate.

The effect of scattering in a real molecular gas is analyzed in [8, 9] using the example of a Doppler line. It is shown that hemispherical emissivity with account for scattering can increase by 32%, depending on the optical thickness of the gas and the probability of quantum survival. In the same works it was found that at an optical thickness of the gas equal to 0.1031 and an optical thickness of the particles with respect to scattering of 0.6510, the radiation of a plane layer with particles is greater that of a layer of a pure gas.

The fact that scattering of radiation on particles of a solid phase leads to large errors in measurements of the temperature of hot gases is confirmed experimentally in [10]. At a small optical thickness, $0.1 < \tau < 0.5$, the error of gas-temperature measurement can reach 100 K.

In all the works mentioned, different models of bands and different methods of solving the equation of radiation transfer of varying accuracy are used. An analysis of the literature showed that so far there are no studies in which a thorough systematic analysis of the effect of the optical properties of the particles on the contour of lines within a wide rage of variation of the parameters of the particles – force, half-width, and distance between them – was made. The authors are familiar with only one paper of a methodological character [9] where an attempt was made to qualitatively analyze different experimental data on estimation of the effect of particles on radiation of a high-temperature gas component. All this stimulated the authors to study the problem mentioned.

The effect of processes of scattering of radiation on particles of a solid phase in a plane layer of a selectively emitting and absorbing gas was analyzed in [11] in detail. This paper deals with the main laws governing the effect of scattering on the contour of the line of emerging radiation. To describe the selective properties of the gas, we used the Elsasser model with the Lorentz contour of the line [1]. The scattering properties of the particles are taken into account by introduction of the Schuster parameter $\mathrm{Sc} = \sigma/(\sigma + s/d)$. Propagation of radiation in a plane layer with transparent boundaries and uniform distribution of sources is studied on the basis of an analysis of the results of a numerical solution of the integro-differential equation of transfer, the method of solution of which is given in [12]. The error of solution of the equation did not exceed 0.001%. It is shown by direct numerical integration of the equation of transfer of radiation over the contour of the line within $-d/2$ and $d/2$ that the processes of scattering on particles deform the contour of the spectral line of emerging radiation. The character of this effect greatly depends on the parameters of the fine structure of the line – its half-width, force, and the Schuster number. An analysis of the obtained results indicates that the mean optical thickness over the contour of the line with account for scattering can be presented in the form

$$\tau\,(\gamma/d,\, s/d,\, \mathrm{Sc},\, L) = \tau_0 \xi\,(\gamma/d,\, \tau_0)\, \vartheta\,(\gamma/d,\, \tau_0,\, \mathrm{Sc})\,, \tag{1}$$

where $\tau_0 = Ls/d$, and ξ and ϑ are correction factors for selectivity and scattering of radiation in the medium, respectively. This representation of the mean (or equivalent) optical thickness makes it possible to avoid integration with respect to the contour of the line and to obtain simple expressions for practical calculations of the characteristics of radiation of selective gases with account for scattering in finite spectral ranges.

The formula of the correction factor $\xi(\gamma/d,\, \tau_0)$, which makes it possible to allow for the effect of the parameters of the fine structure of the lines within a wide range of a change in the parameters is given in [11]. Below we present an analytical expression of the correction factor $\vartheta(\gamma/d,\, \tau_0,\, \mathrm{Sc})$ for accounting for the effect of scattering on the contour of the line of radiation emerging from a layer of a selectively absorbing, radiating, and scattering medium and analyze its error.

TABLE 1. Correction Factor $\vartheta(\gamma/d, \tau, \mathrm{Sc})$ at Different Values of γ/d for the Elsasser Band Model

τ	Sc									
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\gamma/d = 0.001$										
0.01	1.000	0.994	0.989	0.983	0.978	0.973	0.967	0.960	0.948	0.922
0.015	1.000	0.993	0.987	0.981	0.975	0.968	0.961	0.951	0.935	0.900
0.023	1.000	0.992	0.984	0.977	0.969	0.962	0.952	0.939	0.918	0.873
0.036	1.000	0.990	0.981	0.972	0.963	0.954	0.941	0.924	0.898	0.842
0.055	1.000	0.988	0.976	0.966	0.955	0.942	0.926	0.904	0.870	0.809
0.084	1.000	0.984	0.970	0.957	0.943	0.927	0.907	0.880	0.840	0.771
0.128	1.000	0.980	0.963	0.947	0.928	0.908	0.882	0.852	0.805	0.734
0.197	1.000	0.974	0.952	0.930	0.909	0.883	0.855	0.820	0.771	0.698
0.30	1.000	0.967	0.939	0.913	0.885	0.856	0.823	0.781	0.734	0.668
0.46	1.000	0.956	0.921	0.888	0.858	0.825	0.791	0.751	0.703	0.645
0.71	1.000	0.944	0.902	0.864	0.828	0.793	0.755	0.714	0.674	0.639
1.08	1.000	0.927	0.874	0.832	0.796	0.759	0.722	0.689	0.659	0.661
1.66	1.000	0.910	0.849	0.805	0.765	0.728	0.694	0.669	0.656	0.717
2.54	1.000	0.886	0.817	0.771	0.734	0.704	0.685	0.674	0.693	0.813
3.88	1.000	0.862	0.794	0.749	0.717	0.694	0.688	0.700	0.767	0.983
5.95	1.000	0.837	0.762	0.729	0.710	0.707	0.718	0.770	0.889	1.129
9.10	1.000	0.825	0.757	0.736	0.736	0.757	0.807	0.909	1.073	1.235
13.94	1.000	0.806	0.764	0.767	0.802	0.855	0.942	1.074	1.221	1.335
21.33	1.000	0.800	0.794	0.837	0.916	1.011	1.138	1.245	1.337	1.407
32.66	1.000	0.834	0.890	0.969	1.092	1.218	1.316	1.411	1.482	1.487
50.00	1.000	0.916	1.044	1.160	1.300	1.409	1.504	1.594	1.635	1.635
$\gamma/d = 0.01$										
0.01	1.000	0.998	0.995	0.991	0.988	0.984	0.981	0.977	0.972	0.962
0.015	1.000	0.997	0.994	0.990	0.986	0.981	0.976	0.970	0.960	0.946
0.023	1.000	0.997	0.993	0.988	0.983	0.977	0.970	0.962	0.948	0.922
0.036	1.000	0.996	0.991	0.984	0.978	0.970	0.960	0.947	0.928	0.892
0.055	1.000	0.995	0.987	0.979	0.970	0.960	0.946	0.928	0.901	0.852
0.084	1.000	0.993	0.983	0.972	0.960	0.945	0.927	0.902	0.870	0.806
0.128	1.000	0.990	0.977	0.963	0.946	0.928	0.905	0.876	0.835	0.763
0.197	1.000	0.986	0.969	0.950	0.929	0.908	0.879	0.845	0.798	0.727
0.30	1.000	0.982	0.960	0.936	0.913	0.883	0.851	0.815	0.765	0.706
0.46	1.000	0.975	0.947	0.921	0.890	0.859	0.825	0.784	0.742	0.693
0.71	1.000	0.968	0.933	0.899	0.867	0.834	0.800	0.765	0.727	0.710
1.08	1.000	0.960	0.919	0.880	0.844	0.813	0.779	0.752	0.734	0.763
1.66	1.000	0.950	0.906	0.866	0.831	0.798	0.772	0.758	0.769	0.871
2.54	1.000	0.943	0.885	0.852	0.824	0.804	0.797	0.821	0.887	1.031
3.88	1.000	0.938	0.887	0.852	0.835	0.839	0.858	0.917	1.015	1.242
5.95	1.000	0.931	0.894	0.886	0.891	0.922	0.998	1.101	1.243	1.430
9.10	1.000	0.945	0.934	0.959	1.010	1.091	1.185	1.313	1.458	1.653
13.94	1.000	0.969	1.023	1.100	1.187	1.282	1.377	1.491	1.659	1.871
21.33	1.000	1.058	1.193	1.317	1.419	1.489	1.602	1.743	1.894	2.088
32.66	1.000	1.205	1.349	1.468	1.581	1.713	1.845	1.996	2.153	2.268
50.00	1.000	1.390	1.552	1.678	1.777	1.883	2.023	2.165	2.291	2.422
$\gamma/d = 0.1$										
0.01	1.000	1.000	0.999	0.998	0.998	0.997	0.996	0.996	0.996	0.996
0.015	1.000	1.000	0.999	0.998	0.997	0.996	0.996	0.995	0.994	0.994
0.023	1.000	1.000	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.991
0.036	1.000	1.000	0.999	0.997	0.996	0.995	0.993	0.992	0.989	0.987
0.055	1.000	0.999	0.998	0.997	0.995	0.993	0.990	0.988	0.983	0.978
0.084	1.000	0.999	0.997	0.995	0.992	0.989	0.986	0.982	0.976	0.963
0.128	1.000	0.998	0.996	0.992	0.988	0.984	0.979	0.971	0.961	0.946
0.197	1.000	0.997	0.993	0.989	0.983	0.977	0.967	0.957	0.944	0.917
0.30	1.000	0.996	0.991	0.984	0.976	0.966	0.954	0.937	0.917	0.889
0.46	1.000	0.994	0.987	0.977	0.966	0.955	0.940	0.919	0.896	0.874
0.71	1.000	0.993	0.983	0.973	0.959	0.942	0.926	0.908	0.888	0.895
1.08	1.000	0.993	0.981	0.968	0.953	0.941	0.926	0.913	0.921	0.967
1.66	1.000	0.994	0.985	0.975	0.965	0.956	0.956	0.976	1.018	1.096
2.54	1.000	0.997	0.992	0.990	0.998	1.017	1.047	1.107	1.158	1.227
3.88	1.000	1.008	1.023	1.048	1.088	1.141	1.209	1.249	1.297	1.378
5.95	1.000	1.038	1.109	1.181	1.235	1.288	1.343	1.390	1.444	1.531
9.10	1.000	1.142	1.275	1.355	1.381	1.414	1.464	1.520	1.578	1.690
13.94	1.000	1.365	1.435	1.477	1.497	1.533	1.585	1.641	1.725	1.846
21.33	1.000	1.496	1.527	1.566	1.600	1.629	1.680	1.736	1.814	1.927
32.66	1.000	1.543	1.572	1.607	1.636	1.670	1.712	1.758	1.825	1.878
50.00	1.000	1.535	1.554	1.576	1.598	1.624	1.646	1.671	1.696	1.728

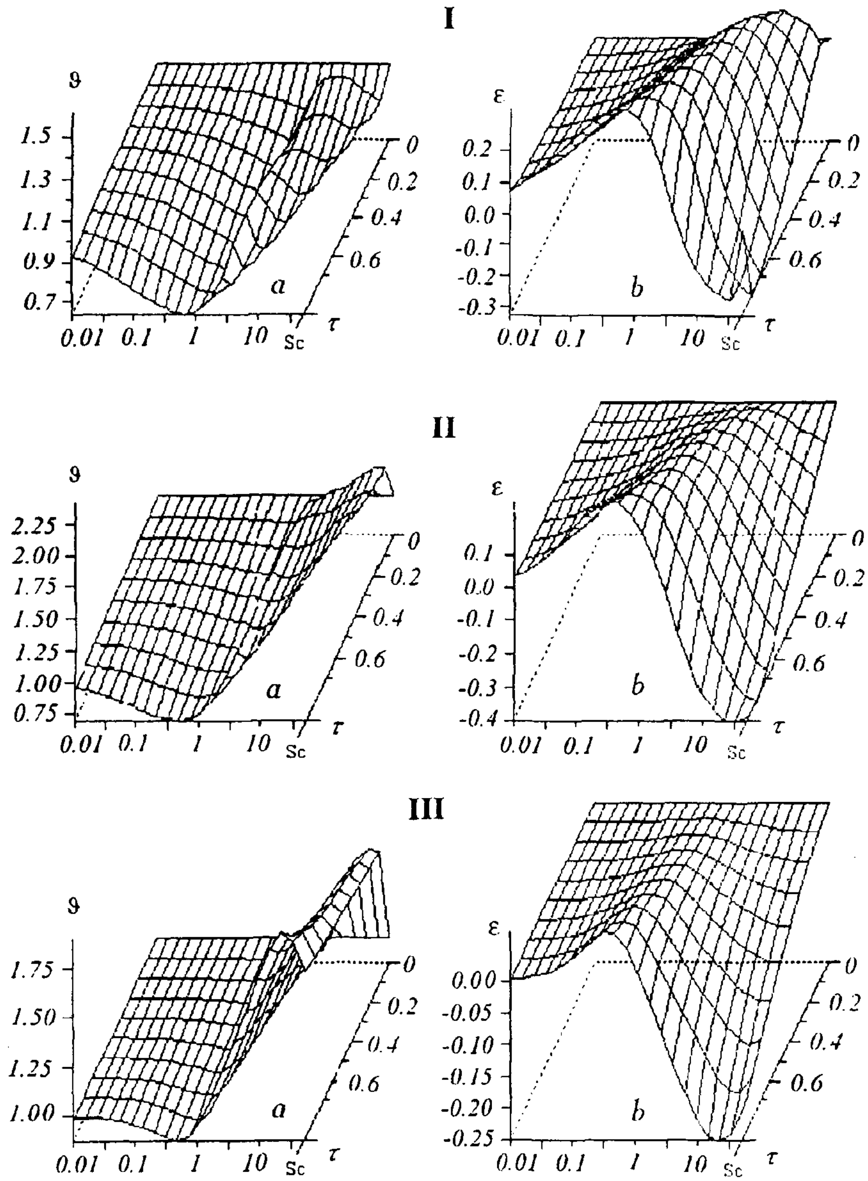

Fig. 1. Distribution of correction factor (a) and errors of calculation of intensity of radiation arising in integration of it (b) at $\gamma/d = 0.001$ ($\varepsilon_{\max} = 0.334$ – I); $\gamma/d = 0.01$ ($\varepsilon_{\max} = 0.402$ – II); $\gamma/d = 0.1$ ($\varepsilon_{\max} = 0.251$ – III).

As in the case of the study of radiation of a selective gas medium, we introduce into consideration the relative error of calculation of the intensity of radiation which arises if the effect of scattering processes on the line contour-mean coefficient of absorption of the selective component ($\vartheta = 1$) is disregarded

$$\varepsilon = 1 - I(\vartheta)/I(\vartheta = 1) . \tag{2}$$

The values of the correction factor $\vartheta(\gamma/d, \tau, \mathrm{Sc})$ and the error of calculation of the intensity of radiation arising in integration of it, which are obtained as a result of direct numerical integration of the integro-differential equation of transfer with respect to the contour of the line from $-d/2$ to $d/2$ within a wide range of variation of radiative and scattering properties of the gas medium, are presented in Table 1 and illustrated by Fig. 1. An analysis of this data shows that

a) the correction factor $\vartheta(\gamma/d, \tau, \mathrm{Sc})$ changes within the range of from 0.6 to 2.5, thus causing variation of the line contour-mean coefficient of absorption of the selective components;

b) neglect of the effect of scattering processes on the line contour-mean coefficient of absorption of the selective component leads to errors of about 40% in calculation of the intensity of radiation. With an increase in the parameter Sc and an increase in τ_0 this error grows. This is due to the fact that at large τ_0 the processes of scattering facilitate the "manifestation" of the contour of emerging radiation compared to a nonscattering medium, in which the contour of the line is virtually absent (i.e., the selective component behaves as a "grey" medium).

The simplest and the most accurate method for determining the correction factor $\vartheta(\gamma/d,\ \tau,\ \mathrm{Sc})$ is interpolation of its exact numerical values. However, for practical calculations it is convenient to have some relatively simple interpolation formula to evaluate the correction factor $\vartheta(\gamma/d,\ \tau,\ \mathrm{Sc})$ quickly and without storage of additional information.

An analysis of the numerical values of the correction factor and the errors of calculation which arise when the effect of scattering processes on the line contour-mean coefficient of absorption of the selective component is neglected (see Fig. 1) made it possible to suggest the following formula for the correction factor

$$\vartheta = \begin{cases} 1 + 0.047A\,(B + 3.7)\,\mathrm{Sc}/(1.48 - \mathrm{Sc})\,, & B \le 1.9\mathrm{Sc}\,, \\ 1 - 0.0625AB + (0.862 - 0.429\ |B + 2|)\,(B + 1.89\mathrm{Sc})\,, & B > 1.9\mathrm{Sc}\,, \end{cases} \tag{3}$$

$$B = \log(\tau/50)\,,\quad A = \log(0.5\gamma/d)\,.$$

In the ranges $0.01 < \tau < 50$ and $0.001 < \gamma/d < 0.5$, the given formula ensures calculation of the intensity of radiation with an error not larger than 12.5%. In this case, the maximum error $\varepsilon_{\max}(\vartheta)$ arising when the accurate values of $\vartheta(\gamma/d,\ \tau,\ \mathrm{Sc})$ are replaced by those calculated by formula (2) decreases as γ/d increases. Even for $\gamma/d > 0.04$ it becomes less than 10%. The structure of the error distribution $\varepsilon(\tau,\mathrm{Sc})$ is given in Fig. 1 for $\gamma/d \in (0.001,\ 0.01,\ 0.1)$.

It should be noted that for narrower ranges of variation of γ/d and τ_0 more accurate formulas can be selected for the value of the correction factor $\vartheta\ (\gamma/d,\ \tau,\ \mathrm{Sc})$, but, in the opinion of the authors, in these cases it is expedient to use the values of the correction factor obtained by interpolation of accurate data given in Table 1.

NOTATION

χ, σ, coefficients of absorption and scattering of medium; Sc, Schuster criterion; $\tau = \chi L$, optical thickness of layer with respect to absorption; L, geometrical thickness of layer; s, strength of absorption line; γ, half-width of absorption line; d, distance between lines.

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